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OPTIMIZING A LIQUID PROPELLANT ROCKET ENGINE WITH AN AUTOMATED COMBUSTOR DESIGN CODE--AUTOCOM

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16. Abstract

A procedure for automatically designing a liquid propellant rocket engine combustion chamber in an optimal fashion is outlined. The procedure is contained in a digital computer code--AUTOCOM. The code is applied to an existing engine, and design modifications are generated which provide a substantial potential payload improvement over the existing design. Computer time requirements for this payload improvement were small, approximately four minutes in the CDC 6600 computer.

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PREFACE

The work described in this report was carried out under Contract NAS 3-13331, "Development of an Automated Combustor Design Procedure." The project manager for this study was Dr. R. J. Priem of Lewis Research Center. Dr. Priem also developed the automated combustor design concept. Mr. D. S. Hague served as Aerophysics Research Corporation project leader for the study. Mr. R. H. Reichel served as principal investigator for propulsion system analysis, and Mr. R. T. Jones served as principal investigator for program development. This report was prepared and edited by Mrs. Jane Yonke.

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by D. S. Hague, R. H. Reichel, R. T. Jones, and C. R. Glatt
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SUMMARY

A digital computer code, AUTOCOM, has been developed as an aide to the liquid rocket engine designer. The code considers the combined effects of engine performance, stability, pressure drop, injector complexity, chamber length, chamber diameter, and mixture ratio characteristics. The code has the ability to automatically define the optimal chamber design recognizing these diverse engine characteristics. An optimum design is generated by means of function minimization techniques operating on an engine rating which measures the actual engine's payload potential loss from a hypothetical ideal combustor which has one hundred per cent of theoretical C* performance, infinite damping rate for all modes of instability, zero pressure drop, zero chamber length, chamber diameter equal to throat diameter, etc.

The code is applied to the optimization of an existing engine. Payload potential is substantially improved by introduction of a series of design perturbations. Computer time required to develop the improved engine is four minutes on the CDC 6600 computer.

1.0 INTRODUCTION

In designing a liquid rocket combustion chamber the engineer must compromise between characteristics such as performance, stability, weight, injector complexity, cost, etc. These *engine characteristics* are not items which are directly controlled by the designer. Instead, they are complicated functions of the independent *design variables* available to the designer, for example, injector hole size, chamber length, etc. To further complicate the problem, frequently there are *several techniques* that can be used to predict how an engine characteristic (such as performance) varies with the independent design variables.

If the engine designer had infinite funds and time available to him, he could design many combustors with different combinations and permutations of the various independent design variables. Engine characteristics could then be calculated for each design with all the available techniques. If the designer had the ability to digest all of this information, he could then select the optimum design for his particular application. The selection would be made, of necessity, on the basis of *weighting factors* applied to both the various engine characteristics and the characteristic values predicted by different techniques.

With limited funds and time, the designer can only examine a few designs, and, because he is familiar with only a few techniques for calculating the various characteristics, he uses only this limited set of techniques to test the acceptability of each design. Using this approach some characteristics are never determined until after the combustor has been built, tested, and often found unacceptable. For example, stability characteristics which are particularly difficult to assess frequently result in an unacceptable engine design. Usually the designs selected in a project are those that are very similar to designs that have been successful in the past. As a result, a design of another group that would be better for a particular application is frequently neglected or ignored. Similarly, when trouble is encountered during the development phase, changes are made to overcome the particular problem using past experience instead of determining which variable or set of variables could be used to overcome the problem with the least sacrifice to other characteristics.

The work performed under the present contract was directed to the development of a generalized computer program to calculate *all the characteristics* of a given combustor design. The program then uses a *perturbation technique* to determine the changes in the design variables that produce the greatest improvement in the rating of the combustor design. The program then follows the path that produces the greatest improvement in the rating to arrive at a combustor design that has the best combination of all variables. This design is called the *optimum combustor design*. The automated combustor design code which generates the optimum combustor design has been given the acronym AUTOCOM.

In any optimization situation, the engineer/designer is ultimately faced with the problem of selecting the *rating* or value function which is to be minimized. In this report the rating of a design is based on a weighted average of all the characteristics of a given combustor. The weighting factors are constants used to obtain both *average characteristics and a rating.* The constants are intended to allow the designer to introduce his views regarding the importance or validity of one technique for obtaining a given characteristic versus another technique for obtaining the same characteristic. For example, if the designer believes that only one technique is valid for predicting the performance of a given design, he will assign a unity value to the weighting factor constants for that specific characteristic, and the constants for all the other performance characteristics will be zero. Similarly, the weighting factor constants in the equation to obtain a single rating for a given combustor design are intended to allow the designer to introduce the relative importance of different types of characteristics, for example, stability versus performance. The weighting factor constants, therefore, *give the designer the same control and flexibility in the computer program as he has in the present "cut-and-try" system.* To establish a base point for the rating system, a hypothetical ideal combustor is given a rating of zero. The hypothetical ideal combustor would have one hundred per cent of theoretical C* performance, infinite damping rate for all modes of instability, zero pressure drop, zero chamber length, chamber diameter equal to the throat diameter, etc.

Specific techniques for obtaining the various combustor characteristics contained in the AUTOCOM code are outlined in Appendix A of this report. The code is written in a modular fashion which permits rapid extension of the combustor characteristic equations. This approach leads to an open ended code capable of future development and extension consistant with the growth of capability in combustor design analysis.

The optimum combustor design procedure is now an operational tool capable of rapid application to practical design problems. This report is primarily intended as a demonstration of the current version of the AUTOCOM code. An existing liquid propellant rocket engine having a well established rating value is studied. An improved design is then automatically generated by the AUTOCOM code, and a significantly better design is developed. The approach followed is outlined in Section 2; Section 3 describes the nominal engine in detail. Section 4 traces the development of the improved design. Conclusions are presented in Section 5, and a self-contained brief outline of the AUTOCOM analysis procedure is presented in Appendix A. Appendix B describes a recently developed multivariable optimization algorithm which is believed to represent a significant improvement over other existing algorithms in terms of the number of design perturbations required to obtain an optimal design. Appendix C presents a list of weighting factor constants used in the development of a combustor rating or value function for the study of the sample engine; the application of these weighting factor constants is discussed in Section 2.

2.0 APPROACH

The AUTOCOM code considers the following major characteristics in the combustor design synthesis:

- I Performance
- II Stability
- III Pressure Drop
- IV Injector Complexity
- V Chamber Length
- VI Chamber Diameter
- VII Mixture Ratio

In view of the uncertainties associated with prediction of combustor design characteristics, each major characteristic is computed as an *average engine characteristic.* Each average characteristic is an appropriate weighted sum of the characteristic value obtained by alternative accepted computation procedures. Each such computation procedure defines a *specific combustor characteristic.* The weighting factors employed in combining a subset of the specific engine characteristics into a particular average engine characteristic may be selected by the user. They thus can be used to reflect user relative confidence in each specific combustor characteristic.

Each specific engine characteristic is a function of the design variables entering into the combustor design procedure. These *combustor design variables* include

1. Diameter of fuel orifices
2. Diameter of oxidizer orifices
3. Number of fuel orifices
4. Number of oxidizer orifices

5. Volume of fuel manifold
6. Volume of oxidizer manifold
7. Length of fuel orifices
8. Length of oxidizer orifices
9. Length of chamber
10. Diameter of chamber
11. Mixture ratio

A subset of these variables defines each specific engine characteristic.

The *combustor rating* provides a single numerical measure of the combustor's capability and is constructed on the basis of a weighted sum of the average engine characteristics. The weighting factors employed in computing the combustor rating are user-defined in the AUTOCOM code. In this note, these weighting factors are based on the impact of each average engine characteristic on vehicle payload capability; they define the payload penalty associated with each characteristic.

The rating function employed in the AUTOCOM code is

$$\begin{aligned}\phi = A_{FI} \cdot F_I^{B_{FI}} &+ A_{FII} \cdot e^{(B_{FII} + C_{FII} \cdot F_{II})} \\ &+ A_{FIII} \cdot F_{III}^{B_{FIII}} + A_{FIV} \cdot F_{IV}^{B_{FIV}} \\ &+ A_{FV} \cdot F_V^{B_{FV}} + A_{FVI} \cdot F_{VI}^{B_{FIV}} \\ &+ A_{FVII} \cdot F_{VII}^{B_{FVII}} \qquad (1)\end{aligned}$$

where

F_I is the average performance characteristic based on C* efficiency and varies from 0 to 100 per cent.

F_{II} is the average stability characteristic based on an equivalent damping rate and varies from $-\infty$ (damps at an infinite rate with time) to $+\infty$ (grows at an infinite rate with time)

F_{III} is the average pressure drop characteristic based on the pressure drop across the injector face and varies from 0 to ∞.

F_{IV} is the average injector complexity characteristic based on the number of injector elements, type of element, injector cavity volume and injector face thickness: varies from 0 to ∞.

F_V is the average length characteristic based on the chamber length from injector to nozzle throat, varies from 0 to ∞.

F_{VI} is the average chamber diameter characteristic based on the chamber diameter at the injector face, varies from 0 to ∞.

F_{VII} is the average propellant mixture ratio characteristic which varies from 0 to ∞.

and the constants A_{FI}, A_{FII}, . . . ,B_{FI},B_{FII},B_{FIII},. . ., B_{FVII}, C_{FII} are weighting factors used to define appropriate measures for combining the average engine characteristics into the final combustor rating.

The average engine characteristics in turn are appropriate weighted averages of the specific combustor characteristics which are computed from well-defined equations and/or curves accepted by the engineering and scientific community. Weighted averages employed in the AUTOCOM code are

$$F_I = \sum_{i=11}^{i=15} [a_{fi}(100 - f_i)] \tag{2}$$

$$F_{II} = \log_e \sum_{i=20}^{i=29} (e^{a_{fi} \cdot f_i}) \tag{3}$$

$$F_{III} = (a_{f31} \cdot f_{31} + a_{f32} \cdot f_{32})/(a_{f31} + a_{f32}) \tag{4}$$

$$F_{IV} = \sum_{i=41}^{i=44} a_{fi} \cdot f_i \tag{5}$$

$$F_V = a_{f51} \cdot f_{51}^{a_{f52}} \tag{6}$$

$$F_{VI} = a_{f61} (f_{61} - 1.0)^{a_{f62}} \tag{7}$$

$$F_{VII} = a_{f71}(b_{f71} - f_{71})^{a_{f72}} \tag{8}$$

Here, the specific combustor characteristics, f_i, are obtained as follows, with definitions given in Appendix A.

f_{11} is the percent mass vaporized of fuel.

f_{12} is the percent mass vaporized of oxidizer.

f_{13} is the C* efficiency determined by the mixing model of NASA.

f_{14} is the C* efficiency determined by the A. D. Little Correlation for Pulsed Combustors.

f_{15} is the C* efficiency determined by the A. D. Little Correlation for Non-Pulsed Combustors.

f_{20} is the chugging decay rate based on the fuel system.

f_{21} is the chugging decay rate based on the oxidizer system.

f_{22} is the stability characteristic based on the A. D. Little Correlation for Pulsed Operation.

f_{23} is the stability characteristic based on the A. D. Little Correlation for Non-Pulsed Operation.

f_{24} is the stability decay rate characteristic based on the stability analysis of Dykema for the fuel.

f_{25} is the stability decay rate characteristic based on the stability analysis of Dykema for the oxidizer.

f_{26} is the stability decay rate characteristic based on the sensitive time lag model for a longitudinal mode.

f_{27} is the stability decay rate characteristic based on the sensitive time lag model for transverse modes.

f_{28} is the stability decay rate characteristic based on the response function approach of NASA Lewis Research Center.

f_{29} is the stability characteristic based on the non-linear stability analysis of NASA Lewis Research Center.

f_{31} is the injector fuel pressure drop characteristic.

f_{32} is the injector oxidizer pressure drop characteristic.

f_{41} is the number of injector fuel plus oxidizer holes characteristic.

f_{42} is the volume of the injector oxidizer dome characteristic.

f_{43} is the volume of the injector fuel dome characteristic.

f_{44} is the length of the injector oxidizer holes characteristic.

f_{45} is the length of the injector fuel holes characteristic.

f_{46} is the injector type complexity characteristic.

f_{51} is the injector length characteristic.

f_{61} is the combustion chamber diameter characteristic.

f_{71} is the propellant mixture ratio characteristic.

The combustor design optimization process is based on minimization of the combustor rating and, hence, the payload penalty. The rating is clearly a function of the combustor design variables, and the weighting factors entering into both the rating equation and the average engine characteristics. In a given computation, these weighting factors are fixed, based on payload impact and degree of confidence in each specific combustor characteristic. It follows that the combustor optimization problem can be formally stated as

$$\phi^* = \text{Min}\ [\phi(\bar{\alpha}_i)] \tag{9}$$

where ϕ is the combustor rating, ϕ^* is the optimal combustor rating, the α_i are the combustor design variables, and $\bar{\alpha}_i$ is the the vector of these design variables. Equation (9) defines a multivariable optimization problem which, due to the non-analytic nature of several specific combustor characteristics, can only be solved by numerical methods, Reference 1. These methods involve repetitive combustor design evaluations using perturbed sets of combustor design variables. By properly organizing the design variable perturbations on the basis of their effect on the combustor rating, the succession of designs generated can be made to converge to the optimal design satisfying Equation (9).

Selection of successive design variable perturbations involves the application of multivariable search techniques. A variety of such search techniques have evolved in recent years. They include elemental one-parameter-at-a-time methods, organized methods which require the evaluation of first- and second-order partial derivatives $\partial\phi/\partial\alpha_i$ and $\partial^2\phi/\partial\alpha_i\partial\alpha_j$ and finally randomized techniques. The AUTOCOM code contains a selection of all three types of search procedures based on the References 1 and 2 optimization program AESOP. The searches may be used separately or in combination at the user's option. Usually a combination of searches will provide more rapid and regular convergence to the optimal design than will the repetitive application of a single search algorithm such as, for example, steepest-descent.

An overall schematic diagram of the AUTOCOM program is presented in Figure 1. The remainder of this note describes the application of AUTOCOM to the optimization of a liquid rocket engine combustor. The procedures employed to insure an adequate numerical model of the design process while controlling elapsed computer time are described in some detail. Convergence from an initial *nominal design* to the final *optimal design* is reported and convergence plots for the combustor rating and each design variable are supplied. An outline of the available specific combustor characteristic computations is presented in Appendix A.

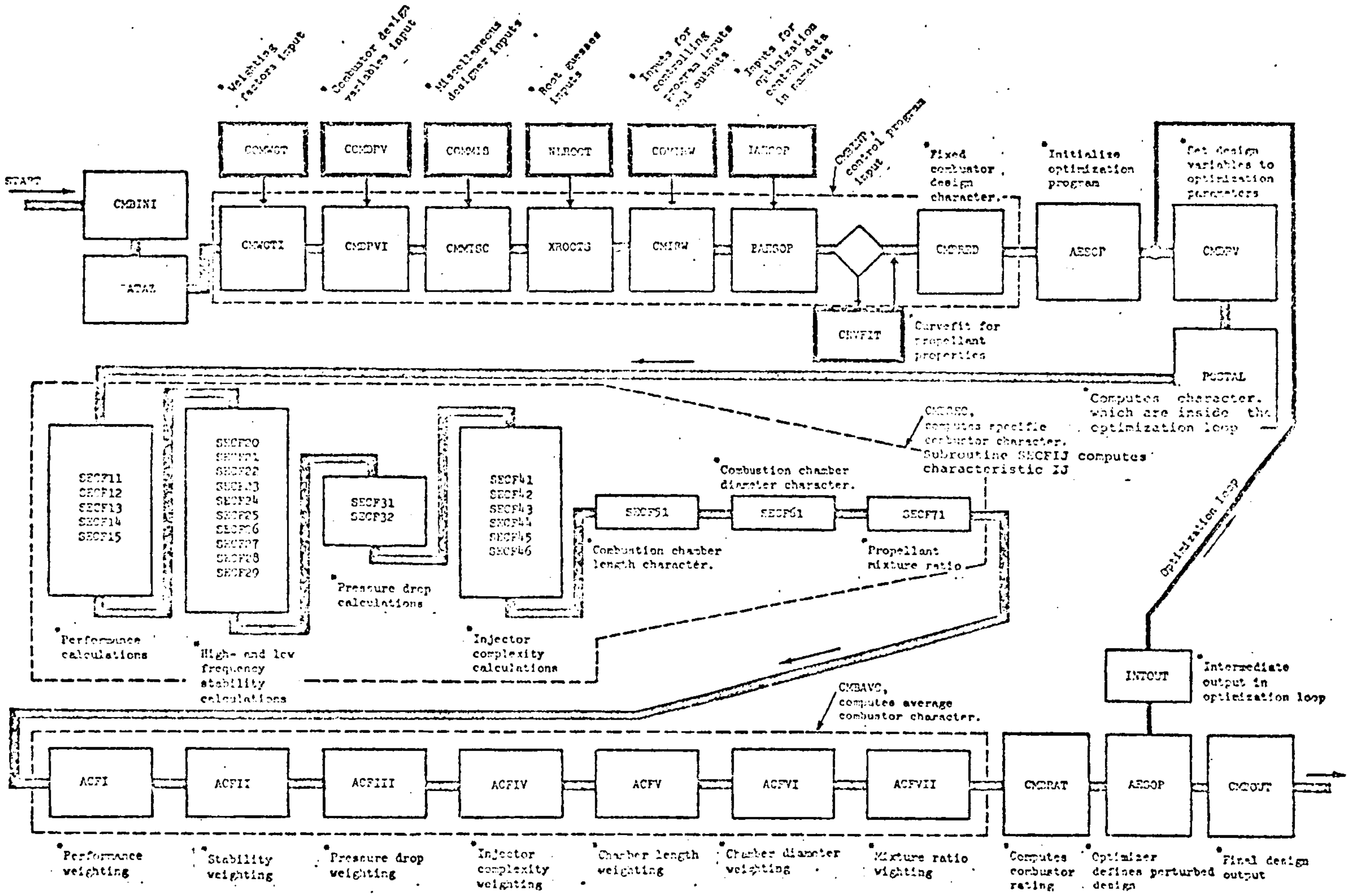


FIGURE 1. OVERALL SCHEMATIC OF AUTOCOM PROGRAM

3.0 THE NOMINAL ENGINE

3.1 DESCRIPTION

The nominal engine is an existing 15000 lbf-thrust liquid rocket engine. Combustor design variables for this engine are

1.	Fuel Orifice Diameter	.195 inches O.D.,.145 inches I.D. (.129 inches diameter equivalent hole)
2.	OX Orifice ΔP Diameter	.084 inches
3.	Number Fuel Elements	216
4.	Number OX Elements	216
5.	Vol. Fuel Manifold	10 $inches^3$
6.	Vol. OX Manifold	10 $inches^3$
7.	Length Fuel Orifices	.06 inches
8.	Length OX Orifices	.40 inches
9.	Length of Chamber	11.22 inches
10.	Diameter Chamber	10.28 inches
11.	Mixture Ratio	5.06 lbm OX/ lbm Fuel

Other pertinent but fixed design parameters include

Propellants	Hydrogen and LOX
Element Type	Concentric Tube (Hydrogen on outside)
OX Orifice Velocity Diam.	.11 inches
Element Impingement Angle	0 degrees
Total Propellant Flow	33.72 lbm/second
Thrust	15000 lbf.
LOX Temperature	Boiling
H_2 Temperature	349°R
Throat Diameter	5.14 inches

This engine runs at the following measured conditions:

Fuel Injection ΔP	83.1 psi
LOX Injection ΔP	48.3 psi
C* Efficiency	98.6 per cent
I_{sp}	444 lbf-sec/lbm
Chamber Pressure (injector face, static)	396.4 psi

These running conditions were used to check the nominal engine description in the combustor synthesis.

Computed running conditions from the AUTOCOM code were

Fuel Injection ΔP	82.9 psi
LOX Injection ΔP	48.5 psi
C* Efficiency	See Section 3.2
I_{sp}	444.8 lbf-sec/lbm
Chamber Pressure (injector face, static)	372.8 psi

It is assumed that the low chamber pressure computed results from the ratio of specific heats employed for the propellant combination (γ = 1.2505) and the combustion temperature (T_c = 5722°R). A more complete set of computed engine running conditions is presented in Table I.

3.2 SPECIFIC COMBUSTOR CHARACTERISTICS

Specific combustor characteristics for the nominal engine are presented in Table II. The average engine characteristics resulting from the selected specific combustor characteristics are shown in Table III. The rating value resulting from the selected average engine characteristic weighting factors is also presented in Table III.

It should be noted that the specific combustor stability characteristic consumes almost all the computational time required for the evaluation of

NUMBER 316

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

GENERAL ENGINE PARAMETERS

COMBUSTOR THRUST FORCE	THRUST = 15000.0
PROPELLANT SPECIFIC IMPULSE	ISP = 444.840
CHAMBER PRESSURE AT INJECTOR HEAD	PCI = 372.765
TOTAL PROPELLANT FLOW RATE	WT = 33.7200
FUEL WEIGHT FLOW RATE	WF = 5.56445
OXIDIZER WEIGHT FLOW RATE	WOX = 28.1556
COMBUSTION TEMPERATURE IN CHAMBER	TCOMB = 5764.91
RATIO OF SPECIFIC HEAT OF COMBUSTION GAS	SPHEAT = 1.24944
GAS CONSTANT OF COMBUSTION GAS (ft.lb./lb.°R)	RGAS = 127.127
IDEAL THRUST COEFFICIENT	CFIDEL = 1.93928
ACOUSTICAL LENGTH OF CHAMBER	LOUM = 8.81574
MOLECULAR WEIGHT OF COMBUSTION GAS	MOLWT = 12.1453
MEAN RESIDENCE TIME OF GAS IN CHAMBER	THETAG = 8.567267E-04
SPEED OF SOUND IN CHAMBER	CS = 65133.9
COMBUSTION CHAMBER MACH NUMBER	MC = .201068
INJECTOR PRESSURE DROP FOR FUEL	DELPF = 82.9049
INJECTOR PRESSURE DROP FOR OXIDIZER	DELPX = 48.4525
COMBUSTION CHAMBER VOLUME	VC = 681.565
AVERAGE VELOCITY OF GASES IN CHAMBER	VELC = 13096.4
FUEL INJECTION VELOCITY	VFUEL = 13013.0
OXIDIZER INJECTION VELOCITY	VOXID = 339.511
TOTAL AREA OF FUEL INJECTOR ORIFICES	TAF = 2.83184
TOTAL AREA OF OXIDIZER INJECTOR ORIFICES	TAX = 1.19702
CHAMBER LENGTH TO VAPORIZE 50 PER-CENT OF FUEL	L5FUEL* = 0.
CHAMBER LENGTH TO VAPORIZE 50 PER-CENT OF OXIDIZER	L5OXID = .636932

* Fuel in a gaseous state

TABLE I. NOMINAL ENGINE RUNNING CONDITIONS
(Dimensions are in inches, lb., sec., °R)

NUMBER 316

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

SPECIFIC COMBUSTOR CHARACTERISTICS

PER CENT MASS FUEL VAPORIZED	F11	= 100. (Gaseous)
PER CENT MASS OF OXIDIZER VAPORIZED	F12	= 99.8122
C* EFFICIENCY MIXING MODEL	F13	= 100.000
C* EFFICIENCY PULSED COMBUSTORS	F14	= 91.3821
C* EFFICIENCY NON-PULSED COMBUSTORS	F15	= 73.7665
FUEL SYSTEM CHUGGING DECAY RATE	F20	= -1036.
OXIDIZER SYSTEM CHUGGING DECAY RATE	F21	= -261.5
PULSED INSTABILITY CHARACTERISTIC	F22	= -37.4819
NON-PULSED INSTABILITY CHARACTERISTIC	F23	= Not Computed
DYKEMA FUEL STABILITY DECAY RATE	F24	= Not Computed
DYKEMA OXIDIZER STABILITY DECAY RATE	F25	= -2964.82
STABILITY LONGITUDINAL TIME LAG	F26	= -1691.31
STABILITY TRANSVERSE TIME LAG	F27	= -412.643
STABILITY LRC RESPONSE FUNCTION	F28	= -360.322
STABILITY PRIEM LINEAR ANALYSIS	F29	= -46427.6
FUEL PRESSURE DROP CHARACTERISTIC	F31	= 82.9049
OXIDIZER PRESSURE DROP CHARACTERISTIC	F32	= 48.4525
FUEL PLUS OXIDIZER HOLES CHARATERISTIC	F41	= 431.000
OXIDIZER DOME VOLUME CHARACTERISTIC	F42	= 10.0000
FUEL DOME VOLUME CHARACTERISTIC	F43	= 10.0000
OXIDIZER HOLE LENGTH CHARACTERISTIC	F44	= .400000
FUEL HOLE LENGTH CHARACTERISTIC	F45	= 6.000000E-02
INJECTOR TYPE COMPLEXITY CHARACTERISTIC	F46	= Not Computed
INJECTOR LENGTH CHARACTERISTIC	F51	= 2.18288
CHAMBER DIAMETER CHARACTERISTIC	F61	= 2.00000
MIXTURE RATIO CHARACTERISTIC	F71	= 5.05990

TABLE II. NOMINAL ENGINE SPECIFIC COMBUSTOR CHARACTERISTICS

```
NUMBER 316                    ENGINE TYPE NO.1
                              TEST CASE
                              THRUST = 15000 POUND
                              O2 / H2 PROPELLANT
                              OPTIMIZATION RUN
```

AVERAGE ENGINE CHARACTERISTICS				RATING COMPONENT
PERFORMANCE CHARACTERISTIC	FI	=	.536361	80.4541
STABILITY CHARACTERISTIC	FII	=	-37.4919	4.71208
PRESSURE DROP CHARACTERISTIC	FIII	=	71.4208	1.49876
INJECTOR COMPLEXITY CHARACTERISTIC	FIV	=	8.37860	3.11377
LENGTH CHARACTERISTIC	FV	=	2.18288	173.002
CHAMBER DIAMETER CHARACTERISTIC	FVI	=	1.00000	35.0000
CHAMBER MIXTURE RATIO CHARACTERISTIC	FVII	=	4.000000E-08	1.729552E-07

```
********************
COMBUSTOR RATING =   297.780
********************
```

TABLE III. NOMINAL ENGINE AVERAGE CHARACTERISTICS AND RATING a combustor. This is due to the time consuming complex characteristic equation solutions required for chugging (f_{20} and f_{21}), longitudinal time lag stability analysis (f_{26}), transverse time lag stability analysis (f_{27}), and the Lewis response function stability analysis (f_{28}). Table IV presents a summary of the characteristic stability equation roots for the nominal engine. The least stable root is obtained from the transverse time lag analysis (f_{27}) using a value of $S_{\nu_h} = 3.0543$.

3.3 A NOTE ON STABILITY ROOTS

Some difficulty was initially experienced in computing the nominal engine stability characteristics for the time lag analyses. The AUTOCOM program assumes a value of the Reardon interaction index, n, of 0.5 for the longitudinal time lag analyses (f_{26}) and 1.0 for the transverse time lag analysis (f_{27}). With these interaction index values, the nominal engine was found to be slightly unstable in two of the transverse modes, Table V. A sensitivity study on the effect of interaction index value was undertaken; as a result, an interaction index value of 0.45 was subsequently utilized in all transverse time lag analyses and an interaction index value of 0.9 was used for the longitudinal analysis. These values were used to obtain the time lag analysis roots shown in Table IV.

A second point should be noted regarding the stability roots. No root is found corresponding to the third value of $S_{\nu_h} = 3.8317$ in the transverse time lag analysis (f_{27}). The missing root can be found by varying the initial guess value in the complex plane for this particular root. Following this procedure the missing root was found to be at the point ($-.59424 \pm j3.5144$) when the Reardon interaction index was 0.9. The root is thus highly damped.

```
NUMBER 316                ENGINE TYPE NO.1
                          TEST CASE
                          THRUST = 15000 POUND
                          O2 / H2 PROPELLANT
                          OPTIMIZATION RUN                                             INTERMEDIATE COMBUSTOR OUTPUT JJJ =      1

             STABILITY ROOT SUMMARY

ROOTS FROM SECF20  (FUEL SYSTEM CHUGGING DECAY RATE)                                  DECAY RATE        FREQUENCY
                                                                                       -.888230          0.
                                                                                       -11.3546          -13.3781
                                                                                       -11.3546           13.3781

ROOTS FROM SECF21  (OXIDIZER SYSTEM CHUGGING DECAY RATE)                              DECAY RATE        FREQUENCY
                                                                                       -.224028           1.59349
                                                                                       -.224028          -1.59349
                                                                                       -.399442          -3.980412E-09

ROOTS FROM SECF26  (STABILITY LONGITUDINAL TIME LAG)                                  DECAY RATE        FREQUENCY
       COAXIAL INJECTION                                                               -.325761          -1.981261E-07
                                                                                       -.227561           3.01091
                                                                                       -.227574          -3.01091
                                                                                       -.962628          -2.335455E-06

ROOTS FROM SECF27  (STABILITY TRANSVERSE TIME LAG)                                    DECAY RATE        FREQUENCY
       COAXIAL INJECTION          IFREQ = 1     SNUH =    1.84129                      -.659403           1.48411
                                                                                       -.659403          -1.48411
       COAXIAL INJECTION          IFREQ = 2     SNUH =    3.05430                    -3.257925E-02        3.05211
                                                                                     -3.257986E-02       -3.05211
       COAXIAL INJECTION          IFREQ = 4     SNUH =    7.01560                    -4.562301E-02        7.02810
                                                                                     -4.580937E-02       -7.02810
       COAXIAL INJECTION          IFREQ = 5     SNUH =    5.33130                    -8.586577E-02        5.28712
                                                                                     -8.587021E-02       -5.28712
       COAXIAL INJECTION          IFREQ = 6     SNUH =    8.52630                      -.466415          -8.74879
       COAXIAL INJECTION          IFREQ = 7     SNUH =    6.70600                      -.248839           6.80011
                                                                                       -.248847          -6.80012

ROOTS FROM SECF28 (STABILITY LRC RESPONSE FUNCTION)                                   DECAY RATE        FREQUENCY
       (1) FIRST TRANSVERSE MODE          BESARG =   1.84120                           -.137018           1.78673
                                                                                       -.137009          -1.78673
       (2) SECOND TRANSVERSE MODE         BESARG =   3.05430                           -.274642           3.29860
                                                                                       -.274604          -3.29859
       (3) THIRD TRANSVERSE MODE          BESARG =   4.20120                           -.273774           4.38764
                                                                                       -.273731          -4.38761
       (4) FIRST RADIAL MODE              BESARG =   3.83170                           -.275702           4.03249
                                                                                       -.275659          -4.03247
       (5) LONGITUDINAL MODE              BESARG =   0.                              -2.843458E-02        2.759602E-02
```

TABLE IV. STABILITY ROOTS IN TIME LAG ANALYSIS

$\left(\begin{array}{ll} f_{26}: & n = .45 \\ f_{27}: & n = .90 \end{array}\right)$

```
NUMBER 315                     ENGINE TYPE NO.1
                               TEST CASE
                               THRUST = 15000 POUND
                               O2 / H2 PROPELLANT
                               OPTIMIZATION RUN

            STABILITY ROOT SUMMARY

ROOTS FROM SECF20  (FUEL SYSTEM CHUGGING DECAY RATE)                              DECAY RATE         FREQUENCY
                                                                                   -.888230           0.
                                                                                   -11.3546          -13.3781
                                                                                   -11.3546           13.3781

ROOTS FROM SECF21  (OXIDIZER SYSTEM CHUGGING DECAY RATE)                          DECAY RATE         FREQUENCY
                                                                                   -.224028           1.59349
                                                                                   -.224028          -1.59349
                                                                                   -.399442          -3.980412E-09

ROOTS FROM SECF26  (STABILITY LONGITUDINAL TIME LAG)                              DECAY RATE         FREQUENCY
       COAXIAL INJECTION                                                           -.346317           9.699875E-08
                                                                                   -.221255           2.99487
                                                                                   -.221247          -2.99486
                                                                                   -.880123          -2.718380E-06

ROOTS FROM SECF27  (STABILITY TRANSVERSE TIME LAG)                                DECAY RATE         FREQUENCY
       COAXIAL INJECTION              IFREQ = 1      SNUH =    1.84129             -.620883           1.48820
                                                                                   -.620883          -1.48820
       COAXIAL INJECTION              IFREQ = 2      SNUH =    3.05430             3.532250E-02       3.05238
                                                                                   3.532753E-02      -3.05238
       COAXIAL INJECTION              IFREQ = 4      SNUH =    7.01560             2.195193E-02       7.02722
                                                                                   2.195308E-02      -7.02722
       COAXIAL INJECTION              IFREQ = 5      SNUH =    5.33130            -2.028810E-02       5.29084
                                                                                  -2.028614E-02      -5.29084
       COAXIAL INJECTION              IFREQ = 6      SNUH =    8.52630             -.421159           8.74290
       COAXIAL INJECTION              IFREQ = 7      SNUH =    6.70600             -.190463           6.79456
                                                                                   -.190456          -6.79456

ROOTS FROM SECF28 (STABILITY LRC RESPONSE FUNCTION)                               DECAY RATE         FREQUENCY
       (1) FIRST TRANSVERSE MODE            BESARG =   1.84120                     -.137018           1.78673
                                                                                   -.137009          -1.78673
       (2) SECOND TRANSVERSE MODE           BESARG =   3.05430                     -.274642           3.29860
                                                                                   -.274604          -3.29859
       (3) THIRD TRANSVERSE MODE            BESARG =   4.20120                     -.273774           4.38764
                                                                                   -.273731          -4.38761
       (4) FIRST RADIAL MODE                BESARG =   3.83170                     -.275702           4.03249
                                                                                   -.275659          -4.03247
       (5) LONGITUDINAL MODE                BESARG =   0.                         -2.843458E-02       2.759602E-02
```

TABLE V. STABILITY ROOTS IN TIME LAG ANALYSIS

$\begin{pmatrix} f_{26}: n = .5 \\ f_{27}: n = 1.0 \end{pmatrix}$

When commencing an optimization study, side analyses of the above type may be required to locate particularly difficult roots. This procedure should also be followed whenever the root imaginary part is not approximately equal to the corresponding value of $S\nu_h$ in the transverse time lag analysis (f_{27}) and the Bessel argument, m, in the Lewis response function analysis (f_{28}). This point is discussed further in Section 4.1.

4.0 OPTIMIZATION COMPUTATIONS

Optimization computations were initially undertaken using all specific combustor characteristics and all stability roots. However, it was noted that the design variable perturbations introduced little change in the computer time consuming stability equation roots. Accordingly the combustor analysis was divided into two classes of computation. These were an *approximate analysis* which considered fewer (possibly none) of the stability roots and a *complete analysis* in which all stability roots were computed. It is emphasized that the approximate analysis is only approximate in that the calculation of the less significant stability roots is omitted. Clearly, by a judicious mix of complete and approximate analyses the total elapsed computer time required for the definition of an optimum engine design can be drastically reduced.

4.1 THE FIRST TWENTY ITERATIONS

Following initial experimentation using all stability roots, the engine was subjected to twenty design iterations using all specific combustor characteristics. An approximate analysis mode was employed which considered only the relatively rapid calculation for the longitudinal time lag analysis (f_{26}) and the transverse time lag analysis (f_{27}) for the single S_{ν_h} value of 3.0543 (the least stable transverse time lag root). This approximate analysis permits both longitudinal and transverse stability characteristics to be monitored.

Initial and final stability roots from this optimization calculation are presented in Table VI. It can be seen that little change has occurred in the stability roots. The trend is to increased stability in the less stable transverse mode and to less stability in the more

A. SELECTED STABILITY ROOTS, NOMINAL ENGINE

NUMBER 318

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

INTERMEDIATE COMBUSTOR OUTPUT JJJ = 1

STABILITY ROOT SUMMARY

ROOTS FROM SECF26 (STABILITY LONGITUDINAL TIME LAG)
COAXIAL INJECTION

DECAY RATE	FREQUENCY
-.325761	5.500834E-08
-.227565	3.01091
-.227564	-3.01092

ROOTS FROM SECF27 (STABILITY TRANSVERSE TIME LAG)
COAXIAL INJECTION IFREQ = 1 SNUH = 3.05430

DECAY RATE	FREQUENCY
-3.257876E-02	3.05211
-1.22075	-1.52265

B. SELECTED STABILITY ROOTS AFTER TWENTY DESIGN PERTURBATIONS

NUMBER 318

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

FINAL COMBUSTOR OUTPUT JJJ=21

STABILITY ROOT SUMMARY

ROOTS FROM SECF26 (STABILITY LONGITUDINAL TIME LAG)
COAXIAL INJECTION

DECAY RATE	FREQUENCY
-.315587	4.288281E-07
-.223750	3.01519
-.223748	-3.01520

ROOTS FROM SECF27 (STABILITY TRANSVERSE TIME LAG)
COAXIAL INJECTION IFREQ = 1 SNUH = 3.05430

DECAY RATE	FREQUENCY
-4.673174E-02	3.06709
-1.24530	-1.57179

TABLE VI. SELECTED STABILITY ROOTS FOR NOMINAL ENGINE AND ENGINE AFTER TWENTY DESIGN PERTURBATIONS

$\begin{pmatrix} f26: & n = .45 \\ f27: & n = .90 \end{pmatrix}$

stable longitudinal mode. It may be noted that the transverse time lag analysis of Table VI considers two solutions to the stability equation with $S_{\nu_h} = 3.0543$. These are the true solution with the frequency approximating S_{ν_h}, and a spurious solution with the frequency approximating $\pi/2$. These spurious solutions with a frequency approximating $\pi/2$ are often encountered in the time lag analysis. If the true solution is not obtained on the nominal engine evaluation and the spurious solution is obtained, the AUTOCOM program will "track" the spurious root. Hence, the analyst must take care to insure that the correct roots are found on the nominal design before embarking on an optimization run. This point is also discussed in Section 3.3.

The engine rating after twenty design perturbations and the corresponding average engine characteristics are presented in Table VII. It can be seen that based on the selected average engine characteristic weights which provide the rating in the form of payload lost, a gain of 23½ pounds payload has resulted when compared to the nominal design of Table III. It can also be seen that the average stability characteristic contribution to the rating is now negligible and that payload is being gained primarily by reduction of the performance characteristic penalty. Pursuing this payload improvement, Table VIII, it can be seen that the performance improvement stems from f_{12}, per cent mass of fuel vaporized, and from slight improvement in C* efficiencies for both pulsed and non-pulsed combustors.

4.2 THE FIRST HUNDRED ITERATIONS

Following the first twenty design iterations discussed in Section 4.1, the optimization problem was restarted without any stability analysis; and 100 successive design perturbations were introduced. A combination of the uniform random ray and pattern searches were employed,

NUMBER 318

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

AVERAGE ENGINE CHARACTERISTICS				RATING COMPONENT
PERFORMANCE CHARACTERISTIC	FI	=	.344513	51.6770
STABILITY CHARACTERISTIC	FII	=	-568.349	4.149190E-23
PRESSURE DROP CHARACTERISTIC	FIII	=	75.3314	1.64971
INJECTOR COMPLEXITY CHARACTERISTIC	FIV	=	8.90284	3.13079
LENGTH CHARACTERISTIC	FV	=	2.20692	176.563
CHAMBER DIAMETER CHARACTERISTIC	FVI	=	1.08429	41.1487
CHAMBER MIXTURE RATIO CHARACTERISTIC	FVII	=	7.097338E-05	1.015815E-03

****

COMBUSTOR RATING = 274.170

****

TABLE VII. RATING AFTER TWENTY DESIGN PERTURBATIONS

NUMBER 318

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

SPECIFIC COMBUSTOR CHARACTERISTICS

PER CENT MASS FUEL VAPORIZED	F11	=	100.
PER CENT MASS OF OXIDIZER VAPORIZED	F12	=	100.000
C* EFFICIENCY MIXING MODEL	F13	=	100.000
C* EFFICIENCY PULSED COMBUSTORS	F14	=	91.7089
C* EFFICIENCY NON-PULSED COMBUSTORS	F15	=	73.8398
FUEL SYSTEM CHUGGING DECAY RATE	F20	=	0.
OXIDIZER SYSTEM CHUGGING DECAY RATE	F21	=	0.
PULSED INSTABILITY CHARACTERISTIC	F22	=	0.
NON-PULSED INSTABILITY CHARACTERISTIC	F23	=	0.
DYKEMA FUEL STABILITY DECAY RATE	F24	=	0.
DYKEMA OXIDIZER STABILITY DECAY RATE	F25	=	0.
STABILITY LONGITUDINAL TIME LAG	F26	=	-1638.16
STABILITY TRANSVERSE TIME LAG	F27	=	-568.349
STABILITY LRC RESPONSE FUNCTION	F28	=	0.
STABILITY PRIEM LINEAR ANALYSIS	F29	=	0.
FUEL PRESSURE DROP CHARACTERISTIC	F31	=	87.9479
OXIDIZER PRESSURE DROP CHARACTERISTIC	F32	=	50.0986
FUEL PLUS OXIDIZER HOLES CHARACTERISTIC	F41	=	433.943
OXIDIZER DOME VOLUME CHARACTERISTIC	F42	=	10.0228
FUEL DOME VOLUME CHARACTERISTIC	F43	=	9.97392
OXIDIZER HOLE LENGTH CHARACTERISTIC	F44	=	.399630
FUEL HOLE LENGTH CHARACTERISTIC	F45	=	6.005603E-02
INJECTOR TYPE COMPLEXITY CHARACTERISTIC	F46	=	Not Completed
INJECTOR LENGTH CHARACTERISTIC	F51	=	2.20692
CHAMBER DIAMETER CHARACTERISTIC	F61	=	2.08429
MIXTURE RATIO CHARACTERISTIC	F71	=	5.05579

Only f_{26} and f_{27} stability roots were computed

TABLE VIII. SPECIFIC COMBUSTOR CHARACTERISTICS AFTER TWENTY DESIGN PERTURBATIONS

Reference 1. The approximate analysis employed completely neglects the stability characteristic. The rationale for this approach was the negligible stability characteristic contribution to the engine rating, Table VII. This table indicates that the stability characteristic affects the rating in the twenty-fourth significant figure. This is well below the accuracy of the CDC 6600 computer which, with sixty bits, is able to provide approximately ten significant decimal figures.

The nominal engine rating without the penalty of all stability characteristics (4.7 pounds, Table III) is 293.1 pounds. After 100 successive design perturbations introduced through the References 1 and 2 multivariable search program, AESOP, the rating is reduced to 210.7 pounds. Rating convergence is illustrated in Figure 2. Convergence behavior of the combustor design variables is illustrated in Figures 3a through 3c. The combustor design variables were allowed to fluctuate by plus or minus twenty-five per cent of the nominal values in this study. Two of the design variables, the chamber diameter and the number of fuel orifices (which equals the number of oxidizer orifices) are practically on the lower and upper bounds permitted in the study.

The final rating and the characteristic components to the rating are presented in Table IX. Final design variable values together with the search limits employed are tabulated in Table X. From Tables III and IX the rating changes associated with each characteristic are seen to be

Performance Characteristic	20.9 lbs., gain
Stability Characteristic	Not considered
Pressure Drop Characteristic	0.14 lbs., gain
Injector Complexity Characteristic	0.72 lbs., loss

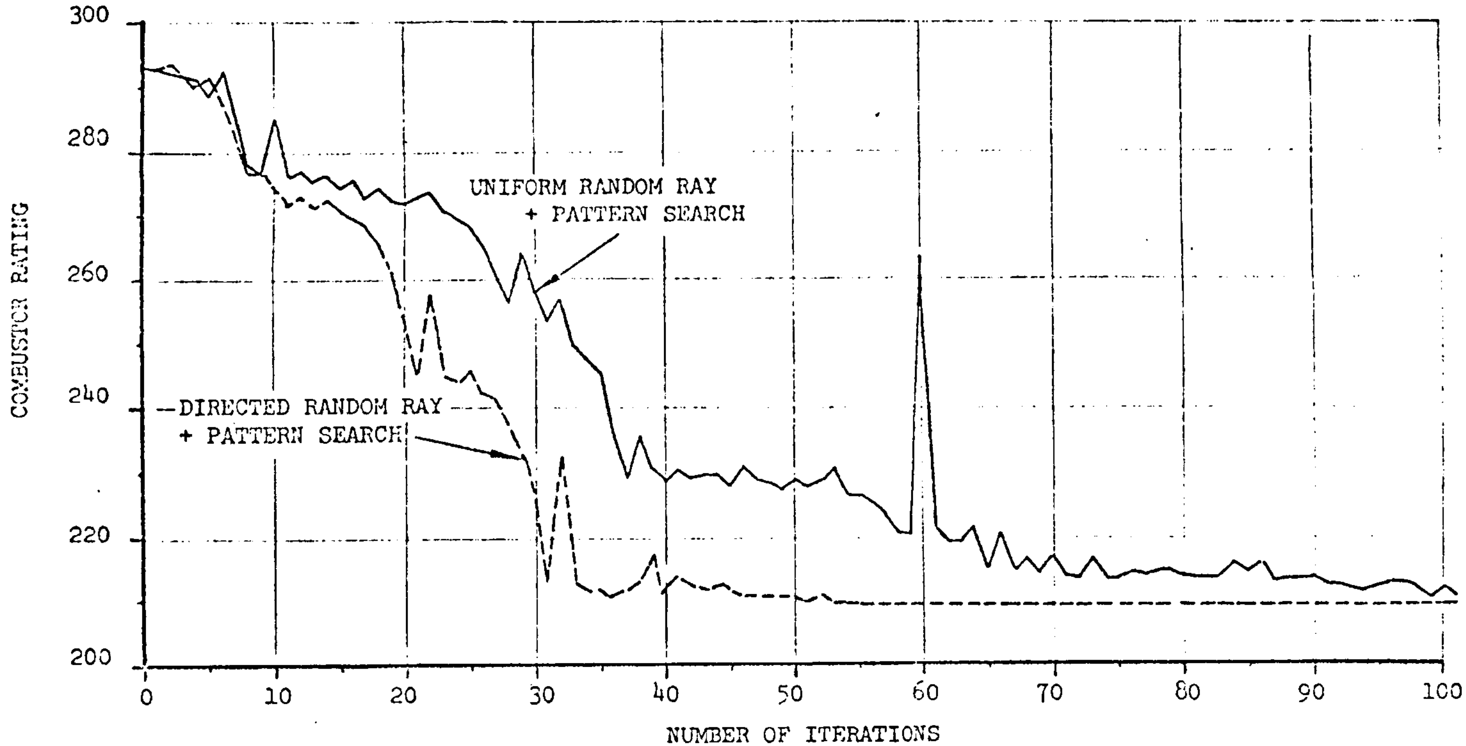
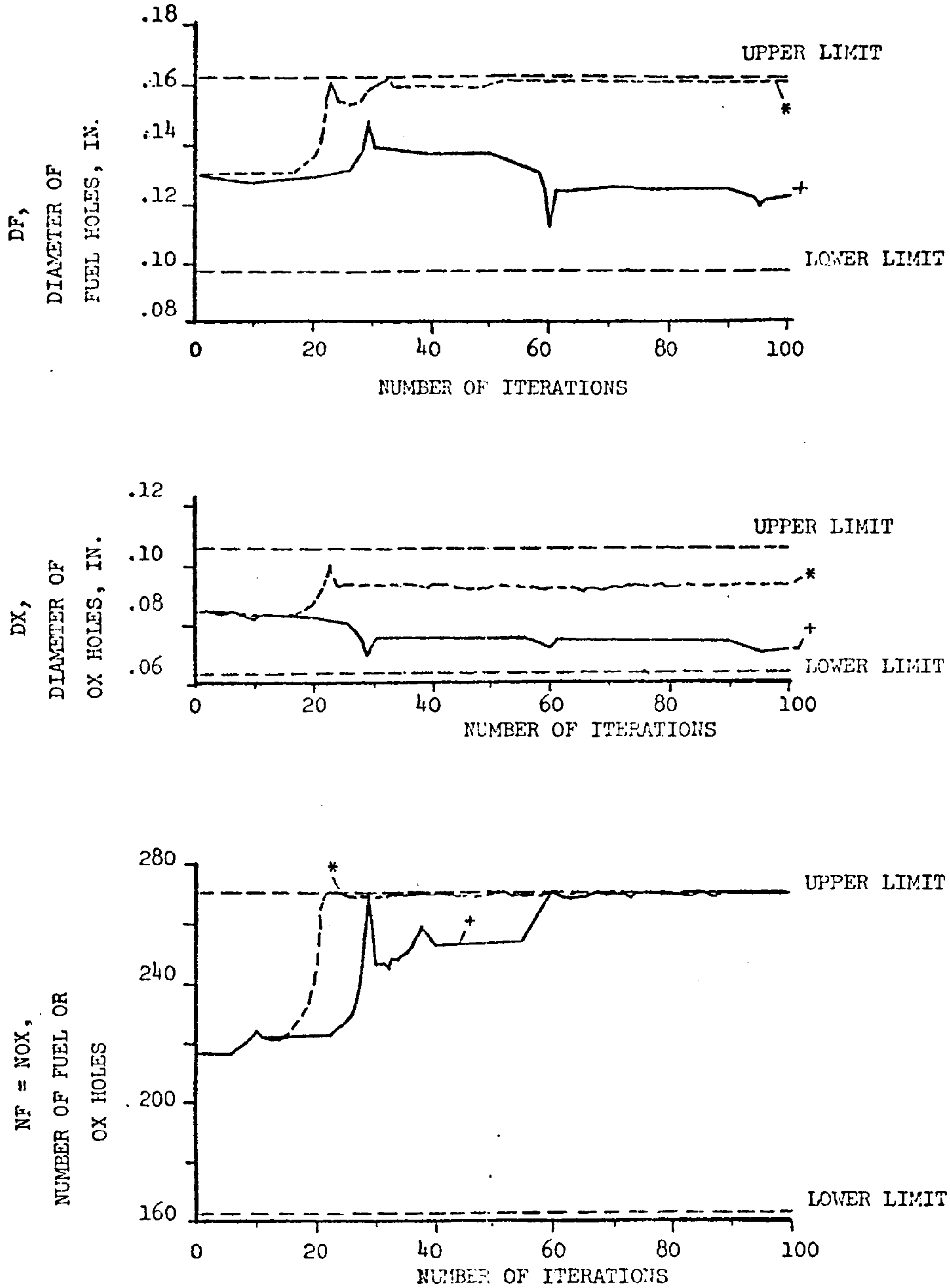


FIGURE 2. COMBUSTOR RATING CONVERGENCE



* Directed random ray + pattern search

\+ Uniform random ray + pattern search

FIGURE 3a

COMBUSTOR DESIGN VARIABLES CONVERGENCE

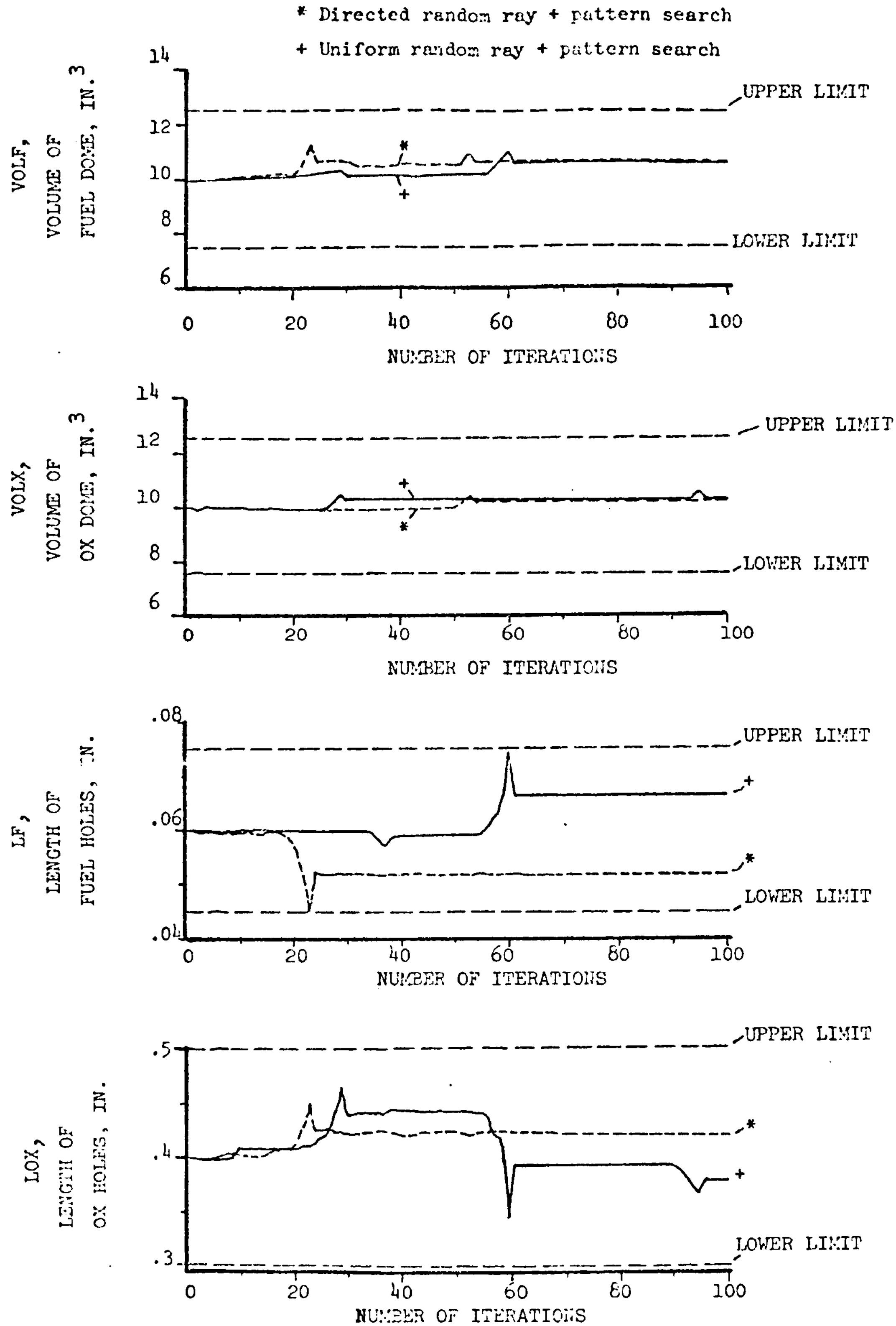


FIGURE 3b. COMBUSTOR DESIGN VARIABLES CONVERGENCE

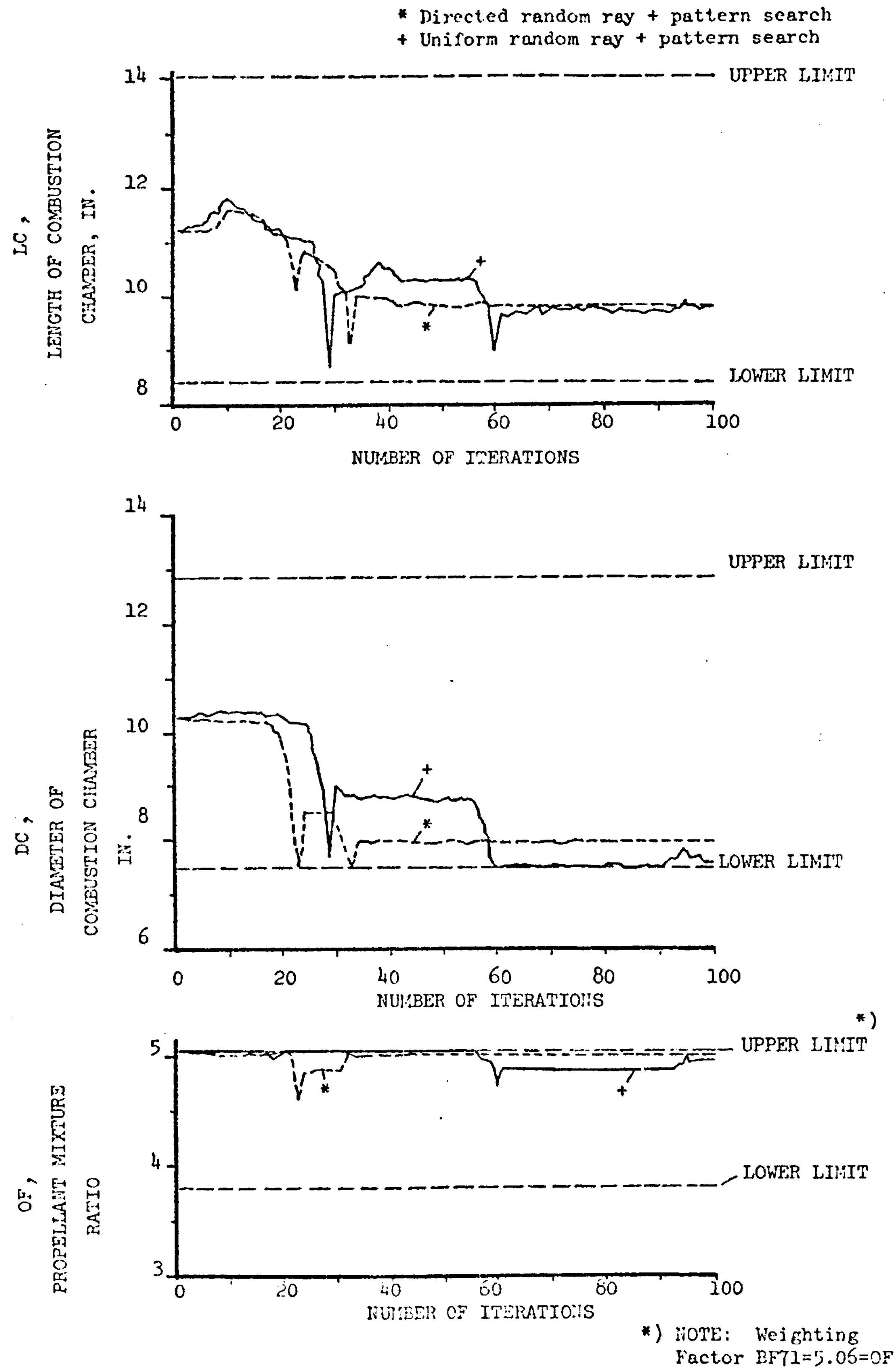


FIGURE 3c. COMBUSTOR DESIGN VARIABLES CONVERGENCE

NUMBER 319

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

AVERAGE ENGINE CHARACTERISTICS				RATING COMPONENT	NOMINAL RATING COMPONENT, TABLE III
PERFORMANCE CHARACTERISTIC	FI	=	.397158	59.5737	80.4541
STABILITY CHARACTERISTIC	FII	=	0.	00.00C	4.71208
PRESSURE DROP CHARACTERISTIC	FIII	=	67.5013	1.35398	1.49876
INJECTOR COMPLEXITY CHARACTERISTIC	FIV	=	9.85680	3.83768	3.11377
LENGTH CHARACTERISTIC	FV	=	1.92038	136.319	173.002
CHAMBER DIAMETER CHARACTERISTIC	FVI	=	.466520	8.28457	35.0000
CHAMBER MIXTURE RATIO CHARACTERISTIC	FVII	=	3.426861E-02	1.31833	1.729552×10^{-7}

****

COMBUSTOR RATING = 210.687

****

NOMINAL COMBUSTOR RATING = 297.780

TABLE IX. RATING AFTER 100 PERTURBATIONS, APPROXIMATE ANALYSIS

NUMBER 319

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

FINAL VALUES OF THE DESIGN VARIABLES

				α High	α Low	Nominal
DIAMETER OF FUEL ORIFICES	DF	=	.121110	.1615	.097	.129
DIAMETER OF OXIDIZER ORFICES	DX	=	7.121544E-02	.105	.063	.084
NUMBER OF FUEL ORIFICES	NF	=	269.257	270.	162.	216.
NUMBER OF OXIDIZER ORFICES	NOX	=	269.257	270.	162.	216.
VOLUME OF FUEL MANIFOLD	VOLF	=	10.5498	12.5	7.5	10.
VOLUME OF OXIDIZER MANIFOLD	VOLX	=	10.3573	12.5	7.5	10.
LENGTH OF FUEL ORIFICES	LF	=	6.653641E-02	.075	.045	.06
LENGTH OF OXIDIZER ORFICES	LOX	=	.378510	.5	.3	.4
ELEMENT TYPE	ETYP	=	1.00000	1.	1.	1.
LENGTH OF CHAMBER	LC	=	9.87075	14.04	8.42	11.22
CHAMBER DIAMETER	DC	=	7.64071	12.86	7.51	10.28
MIXTURE RATIO (OXIDIZER/FUEL)	UF	=	4.96744	5.0599	3.79	5.06

****

FINAL COMBUSTOR RATING = 210.687

****

NOMINAL COMBUSTOR RATING = 297.780

TABLE X. DESIGN VARIABLE VALUES AFTER 100 PERTURBATIONS

Length Characteristic	36.7 lbs., gain
Chamber Diameter Characteristic	26.7 lbs., gain
Chamber Mixture Ratio Characteristic	1.32 lbs., loss
Total Gain	82.4 lbs.

The total rating gain of 82.4 lbs. produced by the optimization process of the AUTOCOM code ignores any stability characteristic effect. To assess this effect, a complete analysis was performed using the Table X vector of combustor design variables. The rating resulting from this complete analysis is presented in Table XI. The associated specific combustor characteristics are presented in Table XII. The stability characteristic produces a rating component of .16 pounds, a 4.55 pound improvement over the nominal engine stability characteristic. Comparing the final rating of 210.84 pounds, Table XI, with the complete nominal engine rating of 297.78 pounds, Table III, the total rating gain obtained in 100 design perturbations is 86.94 pounds. It is interesting to note that despite the use of an approximate analysis which resulted in the stability characteristic being ignored, this characteristic nonetheless improved during the 100 design iterations. Elapsed computer time for the 100 iterations, the final complete analysis, and the initial complete analysis was 250 seconds on the CDC 6600 computer.

4.3 A NOTE ON STABILITY ROOTS AFTER 100 ITERATIONS

The complete stability root set obtained after 100 iterations is presented in Table XIII. It can be seen that the second frequency corresponding to $S_{\nu_h} = 3.0543$ is missing. This root was the least stable on the nominal engine, Table IV, but became more stable in the first 20 iterations of Section 4.1, Table VI. Accordingly, a search for this root was initiated to confirm the stability improvement over 100 iterations. The root was located as a non-conjugate pair at the points

NUMBER 320

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

AVERAGE ENGINE CHARACTERISTICS				RATING COMPONENT	NOMINAL RATING COMPONENT, TABLE III
PERFORMANCE CHARACTERISTIC	FI	=	.397158	59.5737	80.4541
STABILITY CHARACTERISTIC	FII	=	-71.4183	.158260	4.71208
PRESSURE DROP CHARACTERISTIC	FIII	=	67.5011	1.35397	1.49876
INJECTOR COMPLEXITY CHARACTERISTIC	FIV	=	9.85680	3.83768	3.11377
LENGTH CHARACTERISTIC	FV	=	1.92038	136.319	173.002
CHAMBER DIAMETER CHARACTERISTIC	FVI	=	.486519	8.28454	35.0000
CHAMBER MIXTURE RATIO CHARACTERISTIC	FVII	=	3.426941E-02	1.31837	1.729552×10^{-7}

****

COMBUSTOR RATING = 210.845

****

NOMINAL COMBUSTOR RATING = 297.780

TABLE XI. RATING AFTER 100 PERTURBATIONS, COMPLETE ANALYSIS

NUMBER 320

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

SPECIFIC COMBUSTOR CHARACTERISTICS

PER CENT MASS FUEL VAPORIZED	F11	=	100.
PER CENT MASS OF OXIDIZER VAPORIZED	F12	=	100.000
C* EFFICIENCY MIXING MODEL	F13	=	100.000
C* EFFICIENCY PULSED COMBUSTORS	F14	=	87.7264
C* EFFICIENCY NON-PULSED COMBUSTORS	F15	=	72.5578
FUEL SYSTEM CHUGGING DECAY RATE	F20	=	-1264.
OZIDIZER SYSTEM CHUGGING DECAY RATE	F21	=	-184.
PULSED INSTABILITY CHARACTERISTIC	F22	=	-71.4183
NON-PULSED INSTABILITY CHARACTERISTIC	F23	=	Not Computed
DYKEMA FUEL STABILITY DECAY RATE	F24	=	Not Computed
DYKEMA OXIDIZER STABILITY DECAY RATE	F25	=	-2920.49
STABILITY LONGITUDINAL TIME LAG	F26	=	-2861.51
STABILITY TRANSVERSE TIME LAG	F27	=	-840.889
STABILITY LRC RESPONSE FUNCTION	F28	=	-423.008
STABILITY PRIEM LINEAR ANALYSIS	F29	=	-27334.7
FUEL PRESSURE DROP CHARACTERISTIC	F31	=	71.2588
OXIDIZER PRESSURE DROP CHARACTERISTIC	F32	=	59.9856
FUEL PLUS OXIDIZER HOLES CHARACTERISTIC	F41	=	537.514
OXIDIZER DOME VOLUME CHARACTERISTIC	F42	=	10.3573
FUEL DOME VOLUME CHARACTERISTIC	F43	=	10.5468
OXIDIZER HOLE LENGTH CHARACTERISTIC	F44	=	.378510
FUEL HOLE LENGTH CHARACTERISTIC	F45	=	6.653640E-02
INJECTOR TYPE COMPLEXITY CHARACTERISTIC	F46	=	Not Computed
INJECTOR LENGTH CHARACTERISTIC	F51	=	1.92038
CHAMBER DIAMETER CHARACTERISTIC	F61	=	1.48652
MIXTURE RATIO CHARACTERISTIC	F71	=	4.96744

TABLE XII. SPECIFIC COMBUSTOR CHARACTERISTICS AFTER 100 DESIGN PERTURBATIONS

NUMBER 320

ENGINE TYPE NO.1
TEST CASE
THRUST = 15000 POUND
O2 / H2 PROPELLANT
OPTIMIZATION RUN

STABILITY ROOT SUMMARY

			DECAY RATE	FREQUENCY
ROOTS FROM SECF20 (FUEL SYSTEM CHUGGING DECAY RATE)			-.851901	0.
			-8.73039	12.0086
			-8.73039	-12.0086

			DECAY RATE	FREQUENCY
ROOTS FROM SECF21 (OXIDIZER SYSTEM CHUGGING DECAY RATE)			-.339437	2.148328E-07
			-.124060	1.36177
			-.124060	-1.36177

			DECAY RATE	FREQUENCY
ROOTS FROM SECF25 (STABILITY LONGITUDINAL TIME LAG)			-.443645	-3.861826E-07
COAXIAL INJECTION			-.346220	3.01183
			-.346226	-3.01186

			DECAY RATE	FREQUENCY
ROOTS FROM SECF27 (STABILITY TRANSVERSE TIME LAG)				
COAXIAL INJECTION	IFREQ = 1	SNUH = 1.84129	-.340561	-2.56366
COAXIAL INJECTION	IFREQ = 3	SNUH = 3.83170	-5.376885E-02	3.55445
			-5.376337E-02	-3.55499
COAXIAL INJECTION	IFREQ = 4	SNUH = 7.01560	-4.910274E-02	7.03408
			-4.909511E-02	-7.03407
COAXIAL INJECTION	IFREQ = 5	SNUH = 5.33130	-9.847667E-02	5.38269
			-9.847565E-02	-5.38259
COAXIAL INJECTION	IFREQ = 7	SNUH = 6.70600	-.266044	6.35468
			-.266041	-6.35468

			DECAY RATE	FREQUENCY
ROOTS FROM SECF28 (STABILITY LPC RESPONSE FUNCTION)				
(1) FIRST TRANSVERSE MODE	BESARG = 1.84120		-.133496	1.79003
			-.133488	-1.79002
(2) SECOND TRANSVERSE MODE	BESARG = 3.05430		-.266868	3.23059
			-.266829	-3.23056
(3) THIRD TRANSVERSE MODE	BESARG = 4.20120		-.244308	4.34421
			-.244291	-4.34415
(4) FIRST RADIAL MODE	BESARG = 3.83170		-.254212	3.98426
			-.254184	-3.98422
(5) LONGITUDINAL MODE	BESARG = 0.		-2.470107E-02	2.314376E-02

TABLE XIII. STABILITY ROOT SET AFTER 100 ITERATIONS

$$z_1 = (-.443094 + j2.73775)$$

and $$z_2 = (-.552853 + j3.61289)$$

Both roots are well damped; however, since the imaginary parts of these roots differs markedly from the value of S_{ν_h}(3.0543) a "ray search" was carried out through the design space. This search proceeded along the ray joining the nominal engine design to the final design obtained after 100 iterations. The ability to carry out this type of ray search through an n-dimensional space (in this case, a twelve-dimensional space) is a standard feature of the AESOP program. Fifty-two points were equi-distributed along the ray search joining the nominal and final design. The root corresponding to S_{ν_h} = 3.0543 was *tracked along the ray*, starting from the nominal design. Root variation along the ray is presented in Figure 4. The root at

$$z = (-.03257 + j3.0521)$$

presented in Table VI tracks continually into the root at

$$z_1 = (-.443094 + j2.73775)$$

confirming this root as a valid solution to the stability root characteristic equation. Both final roots, z_1 and z_2, obtained for S_{ν_h} = 3.0543 are, therefore, considered to be valid roots. Their heavily dampeu nature results in their providing no contribution to the final engine rating. It can be seen from Figure 4 that the root at z_1 is becoming more stable as the design progresses and that the root at z_2 is becoming less stable.

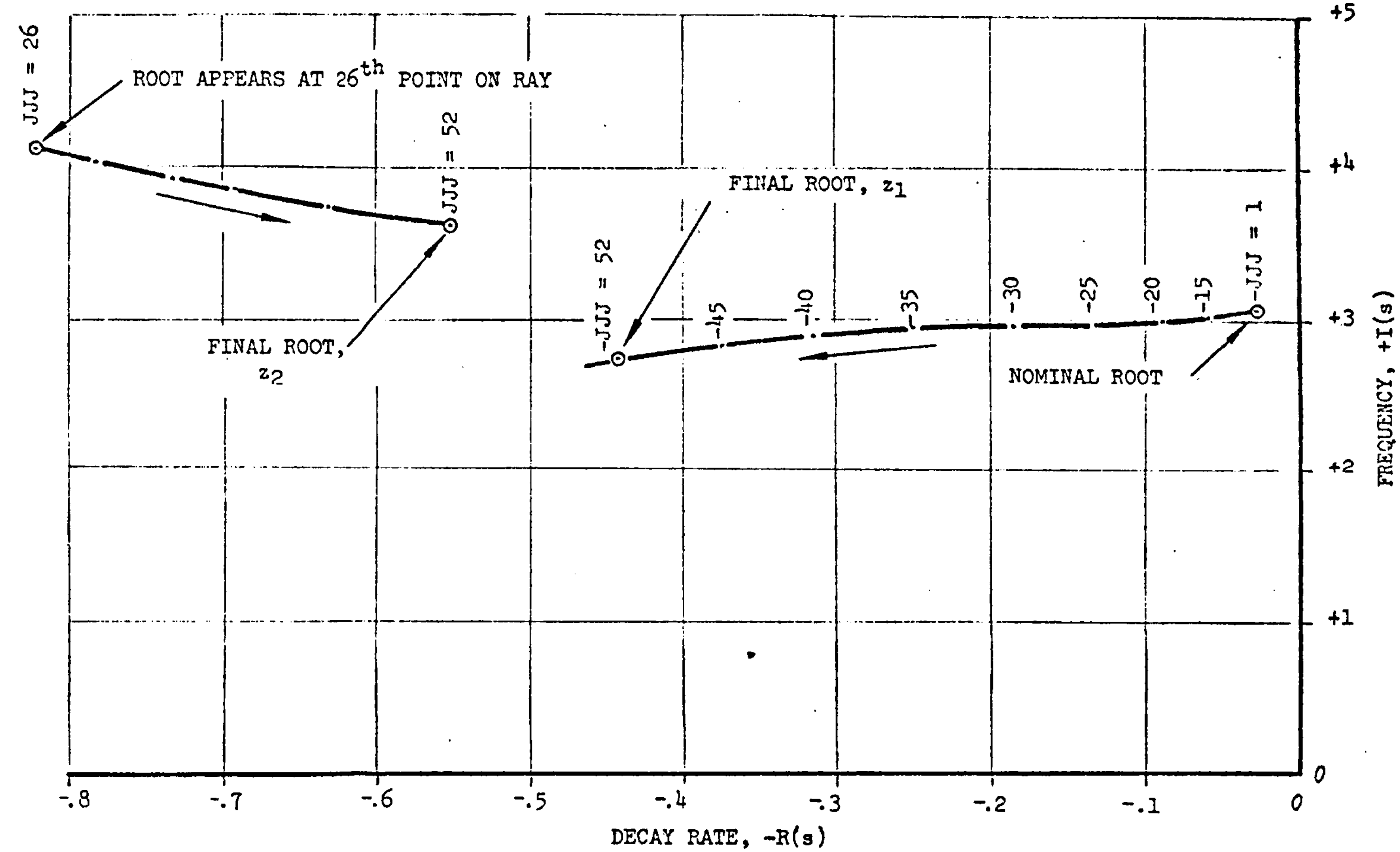


FIGURE 4. ROOT LOCUS PLOT FOR RAY SEARCH

4.4 VERIFICATION OF THE OPTIMAL SOLUTION

The optimal solution reported in Section 4.2 was verified in two ways. *First,* the solution was continued for 100 additional iterations with the uniform, random ray and pattern search algorithm. A slight performance improvement resulted. A final rating of 209.43 pounds was attained, a gain of 1.3 pounds over the solution of Section 2.2. *Second,* the solution was restarted from the nominal solution using a different search algorithm. The algorithm used in this second solution was a recently developed directed random ray search, Appendix B, in combination with the pattern acceleration algorithm. The final rating obtained by this method was 209.46 pounds after 100 iterations. Convergence of this solution has been added to Figure 2. It is clear that a final solution has been obtained. It is also clear that the newly developed search provides more rapid convergence to the solution than the older uniform directed ray search. This behavior is in keeping with other tests of the new search.

5.0 CONCLUSION

The AUTOCOM code has successfully developed an improved engine design starting from the existing nominal engine. The payload potential of the engine was improved by 87 pounds as measured by the rating equation. Computer time required by the AUTOCOM code was minimal. The average time requirement for an assessment of each combustor design was approximately two seconds on the CDC 6600 computer. Computer time absorbed by the optimization subprogram AESOP in determining suitable design variable perturbations was negligible--approximately 103 seconds. The engine was optimized in one hundred design perturbations; hence, total computer time required to optimize the design was approximately four (4) minutes. More computer time would be required if combustor stability problems had been encountered. In this eventuality, it is estimated that twenty (20) minutes computer time would be required to obtain a solution. A definitive assessment of computer time in such a case awaits further experience using the AUTOCOM code.

An examination of the optimal engine components reveals that the payload gain was largely obtained from improvements in the performance, chamber length, and chamber diameter characteristics. Small payload gains also resulted from improved stability and pressure drop characteristics. The injector complexity characteristic and the chamber mixture ratio characteristics both contributed performance losses when the final engine is compared to the nominal engine.

A complicated set of design variable perturbations were introduced to obtain the payload capability improvement. An assessment of the design variable changes introduced by the optimization algorithms indicates that the number of fuel and oxidizer holes, volume of the oxidizer dome, volume of the fuel dome, length of the combustion chamber, chamber diameter, and the mixture ratio are all sensitive design variables in the engine considered. In particular, in both optimal solutions obtained the number of fuel and oxidizer holes rapidly rises to the upper limit permitted, indicating that further payload improvement might result from a further increase in the number of holes allowed. Diameter of the fuel holes, diameter of the oxidizer holes, length of the fuel holes, and length of the oxidizer holes were relatively insensitive design variables for the engine design considered, presumably because of the basic stability of this engine.

6.0 REFERENCES

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APPENDIX A
OUTLINE OF THE AUTOCOM PROGRAM ANALYSIS

The AUTOCOM program automatically determines the combustor chamber characteristics given the chamber design variable values. The analysis considers performance, stability, and injector complexity characteristics. In an optional mode of operation, the program possesses the ability to automatically perturb the design parameters defining the engine characteristics (optimization). Stability and performance analysis modules available within the program are described below.

A1. PER CENT MASS OF FUEL VAPORIZED (FUNCTION f_{11})

Per cent mass of fuel vaporized is computed by the method of NASA TR-67, Reference A1.

> "A model and theory for describing the rocket combustion process are described. The model is based on the assumption that propellant vaporization is the rate-controlling combustion process. Calculations of the vaporization rate and histories show the effects of propellants, spray conditions, engine design parameters and operating parameters on the vaporization process. The results are correlated with an effective chamber length for ease in using them for design purposes. An analysis is presented on the quantitative effect of incomplete propellant vaporization on combustor performance. With this analysis, experimental and calculated combustor performances are compared for injectors where drop size can be calculated. For other injectors the drop sizes are deduced and are shown as functions of injector type and orifice size."

A2. PER CENT MASS OF OXIDIZER VAPORIZED (FUNCTION f_{12})

Per cent mass of oxidizer vaporized is computed by the method of Reference A1 in a similar manner to the fuel vaporization method summarized above.

A3. C* EFFICIENCY BY MIXING MODEL (FUNCTION f_{13})

The first method available for computing C* efficiency is based on the method of NASA TN-2881, Reference A2:

> "A model for predicting rocket combustion performance is presented which is based on the assumption that performance is limited only by gas-phase turbulent diffusion, or mixing, of oxidant and fuel vapors. The model shows how mixture ratio, chamber length, injector-hole spacing, and turbulence intensity affect performance.
>
> "Many physical processes occur simultaneously in a rocket combustor. In order to understand the importance of the various processes, such as vaporization, gas-phase mixing, or chemical reaction, each one is considered separately so that their effects on combustor performance may be determined and compared. The vaporization process in rocket combustion is well understood, and an exhaustive analysis of it has been presented in the literature. Chemical reaction rates are usually considered to be very rapid and, therefore, not a limiting factor in controlling the rocket combustion process. A treatment of the relative importance of chemical reaction rates in rocket combustion is presented in Reference A3. The mixing process, though less understood, may possibly be, under certain conditions, a rate limiting step in the combustion process.

"In essence, the model developed in Reference A2 combines the highly generalized results of Reference A4 with a technique similar to that suggested in Reference A1. In Reference A1, it is suggested that the effect of mixing on performance may be determined by calculating the performance of many small areas in a combustor cross section and averaging the results. The results of Reference A4 show how propellant concentration varies radially across the combustor as a function of chamber length, injector hole spacing, and intensity of turbulence, but do not indicate what effect such variations might have on combustor performance.

"The method of NASA TN-2881 translates the generalized concentration profiles of Reference A4 into combustor performance values. A model based on that of Reference A4 enables mixing-limited performance to be calculated for particular propellant systems as a function of chamber length, turbulence intensity, injector-hole spacing, and operating propellant mixture ratio. Results of detailed digital computer calculations using this model are presented in Reference A2 for eight propellant systems: oxygen with hydrogen, ammonia, hydrazine, and JP-4; fluorine with hydrogen, ammonia, and hydrazine; and nitrogen tetroxide with hydrazine."

A4. C* EFFICIENCY FOR PULSED COMBUSTORS (FUNCTION f_{14})

C* efficiency for pulsed combustors is computed by the statistical relationships presented in NASA CR-72370, Reference A5.

"The objective of this method is the establishment of criteria for the design of stably operating liquid propellant rocket engines by means of a systematic analysis of existing test data. In this analysis, relationships were sought between engine design variables, operating variables, and stability characteristics. The results of theoretical and experimental studies of combustion instability were used as guides in seeking these relationships.

"The method was established by

1. Development of a system for collecting rocket engine stability test data and utilization of this system to collect such data from a wide variety of engines.

2. Definition and evaluation of functions of engine variables (parameters) which may be related to stability characteristics.

3. Establishment of relationships between engine design and stability parameters by analysis of the collected experimental data.

4. Formulation of an approach for utilizing these design-stability relationships in the development of new engines.

"The results provide a comprehensive description of past experience with combustion instability in various engine types. The design approach offers a means for utilizing this experience to avoid development of new engines which are prone to instability."

A5. C* EFFICIENCY OF NON-PULSED COMBUSTORS (FUNCTION f_{15})

C* efficiency for non-pulsed combustors is computed by the statistical relationships presented in NASA CR-72370, Reference A5, discussed above.

A6. CHUGGING DECAY RATE BASED ON FUEL SYSTEM (FUNCTION f_{20})

Function f_{20} measures the fuel system chugging decay rate based on the method of Reference A6 for either pump or pressure fed systems. Pump fed systems decay rates are found from the eigenvalues of the characteristic equation 2.04.03 of Reference A6.

$$F(s) = (1 + Es + JEs^2)[1 + s - n + ne^{-\bar{\tau}s}] + PEse^{-\bar{\tau}s} = 0 \qquad (2.04.03)$$

Pressure fed system decay rates are found from the eigenvalues of the characteristic equation (2.05.02) of Reference A6.

$$F(s) = [1 + Js + JEys^2 + J^2Ey(1-y)s^3](1 + s - n + ne^{-\bar{\tau}s})$$

$$+ Pe^{-\bar{\tau}s}(1 + JEys^2) = 0 \qquad (2.05.02)$$

Eigenvalues are found by application of modern optimization procedures to minimization of $|F(s)|$ followed by root sweeping.

A7. CHUGGING DECAY RATE BASED ON OXIDIZER SYSTEM (FUNCTION f_{21})

Function f_{21} measures the oxidizer system chugging decay rate by the method of Section A6 above.

A8. STABILITY CHARACTERISTICS FOR PULSED OPERATION BASED ON STATISTICAL CORRELATION (FUNCTION f_{22})

Function f_{22} is the characteristic for pulsed operation based on the regression analysis of Reference A5. Basis of this approach is described in Section A4.

A9. STABILITY CHARACTERISTIC FOR NON-PULSED OPERATION BASED ON STATISTICAL CORRELATION (FUNCTION f_{23})

Function f_{23} is the characteristic for non-pulsed operation based on the regression analysis of Reference A5. Basis of the approach is described in Section A4.

A10. FUEL SYSTEM HIGH FREQUENCY STABILITY DECAY RATE BASED ON THE METHOD OF DYKEMA (FUNCTION f_{24})

Function f_{24} is the fuel system high frequency stability decay rate based on the method of Dykema, Reference A7. The characteristic decay rates of selected longitudinal and transverse mode combinations are computed. The Dykema method provides

> "A simplified engineering approach to the analysis of high frequency combustion instability in large liquid rocket engines. The approach stems from theoretical consideration of pressure and time dependent droplet combustion. There results a dimensionless correlating parameter called a stability number (N_S) which essentially represents the dimensionless ratio of a characteristic molecular diffusion time to a characteristic acoustic time. Stable and unstable ranges of N_S are defined, and N_S is reduced and simplified to common, readily measurable engineering terms involving the injector orifice pattern (size and number of orifices), the frequency of the acoustic modes, chamber pressure, and propellant flow rate."

A11. OXIDIZER HIGH FREQUENCY STABILITY DECAY RATE BASED ON THE METHOD OF DYKEMA (FUNCTION f_{25})

Function f_{25} is the high frequency stability decay rate based on the method of Dykema, Reference A7. The characteristic decay rates of selected combinations of longitudinal and transverse modes are computed. The Dykema method is summarized in Section A10.

A12. STABILITY DECAY RATE BASED ON SENSITIVE TIME LAG MODEL FOR LONGITUDINAL MODE (FUNCTION f_{26})

Function f_{26} is the stability decay rate characteristic based on the sensitive time lag model for a longitudinal mode, Reference A6. The correlation equations for the interaction index developed by Reardon of Aerojet is incorporated in the model. Decay rates are based on the eigenvalues of Equation (3.01.20) of Reference A6.

$$\frac{1-Be^{2s}}{1+Be^{2s}} = M[(1 - \gamma n) + \gamma n e^{-\bar{\tau} s}] \qquad (3.01.20)$$

The solution is subdivided into

a. Non-hypergolic propellant with coaxial injection
b. Non-hypergolic propellant with non-coaxial injection
c. Storable propellants

A13. STABILITY DECAY RATE BASED ON SENSITIVE TIME LAG MODEL FOR TRANSVERSE MODE (FUNCTION f_{27})

Function f_{27} is the stability decay rate characteristic based on the sensitive time lag model of Reference A8. Decay rates are based on modifications to the characteristic Equations (28) of Reference A8.

$$h_1 P + h_2 = 0$$

$$h_1 = \gamma \bar{u}_e [1 - js_{\nu_h} E \int_0^{z_e} (\bar{u}/\bar{u}_e) dz]$$

$$h_2 = -(\gamma + 1)\bar{u}_e - j(f - \frac{1}{f})s_{\nu_h} z_e$$

$$+E[\frac{1}{f} - \frac{s_{\nu h} z_e^2}{2} (f - \frac{1}{f}) + j(\gamma + 1)s_{\nu_h}\bar{u}_e \int_0^{z_e} (\bar{u}/\bar{u}_e)dz]$$

As in Function f_{26} the model is specialized for

a. Non-hypergolic propellant with coaxial injection
b. Non-hypergolic propellant with non-coaxial injection
c. Storable propellants

A14. STABILITY DECAY RATE BASED ON THE RESPONSE FUNCTION APPROACH OF LEWIS RESEARCH CENTER (FUNCTION f_{28})

Function f_{28} is the decay rate determined from the acoustic wave solutions of Priem and Rice, Reference A9. Response functions for liquid propellants are determined by Reference A10. Response functions for gaseous propellants are determined by Reference A11.

A15. STABILITY CHARACTERISTIC BASED ON THE *NON-LINEAR ANALYSIS OF PRIEM AND GUENTERT* (FUNCTION f_{29})

Function f_{29} is the decay rate based on the non-linear analysis of Priem and Guentert, Reference A12.

> "Regions of combustion instability in rockets are calculated from a non-linear theory that considered the combustor to be an annular section with very small thickness and length. Two models are used to determine the local burning rate. One assumes that the burning rate is equal to the vaporization rate; the other assumes that the burning rate is equal to the chemical-reaction rate. The results show that a finite disturbance is required to produce instability. The instability regions are found to be a function of several design parameters and to be insensitive to the

activation energy, specific-heat ratio, and order of reaction of the propellants. The vaporization rate model is more sensitive to a pressure disturbance for design parameters corresponding to conditions encountered in large combustors. The chemical-reaction-rate model is more sensitive to a pressure disturbance for conditions corresponding to small research combustors. Wave shapes and characteristics are determined for various conditions."

A16. ENGINE DESIGN AND COMPLEXITY CHARACTERISTICS

The remaining functions in Figure A1 are straightforward engine design and complexity factors.

f_{31} is the fuel pressure drop characteristic

f_{32} is the oxidizer pressure drop characteristic

f_{41} is the number of fuel plus oxidizer holes characteristic

f_{42} is the volume of the oxidizer dome characteristic

f_{43} is the volume of the fuel dome characteristic

f_{44} is the length of the oxidizer holes characteristic

f_{45} is the length of the fuel holes characteristic

f_{46} is an injector-type complexity characteristic

f_{51} is the chamber length characteristic

f_{61} is the chamber diameter characteristic

f_{71} is the mixture ratio characteristic

APPENDIX A - REFERENCES

A1. Priem, Richard J. and Heidmann, Marcus F.: Propellant Vaporization as a Design Criterion for Rocket-Engine Combustion Chambers. NASA Technical Report R-67, 1960.

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APPENDIX B

DIRECTED RANDOM RAY SEARCH

This search proceeds along a succession of random rays distributed about a best estimate of the gradient vector. The search can be used in combination with the pattern search acceleration procedure of the AESOP program.

The best gradient vector estimate, $\bar{R}$, is based on a weighted combination of the old gradient vector estimate, $\bar{R}_{old}$, and the latest search step direction which improved performance, $\bar{R}'$.

$$R_i = (W_R \cdot R_{i_{old}} + R_i')/(W_R + 1.0) \qquad \text{(B1)}$$

The search step directions explored are based on a weighted combination of the best gradient vector estimate, $\bar{R}$, and a small random vector, r.

$$\delta\alpha_i = (U_R \cdot R_i + r_i)/(U_R + 1) \qquad \text{(B2)}$$

On problems involving a pronounced ridge in the control space, this search will prove efficient. Once the approximate direction of the ridge is established by a performance improvement, the random rays are focused in the general direction of the ridge, and excursions outside the region of improvement tend to be minimized. The search is sensitive to the weighting constant values U_R and W_R. Based on a study of the Rozenbrock Valley problem, nominal values of $W_R = 5.0$ and $U_R = 2.5$ are recommended.

It should be noted that while the directed random ray search proves effective when an approximate ridge direction is defined, Figure B1, it may prove wholly ineffective when the ridge abruptly changes direction, Figure B2, or when acquisition or a ridge requires a large directional change, Figure B3. To avoid convergence failure in these last two situations, the weighting constant, U_r, which focuses the random rays must be adaptively determined. When further progress proves impossible for a given value of U_R, this weighting constant must be decreased. As $U_r \to 0$, the search approaches the uniform random ray search which permits an abrupt change of search direction. Following establishment of a new search direction, the random rays are refocused along the new approximate ridge direction by an increase in U_R. Logic to focus and defocus the directed random rays is included in the AESOP code.

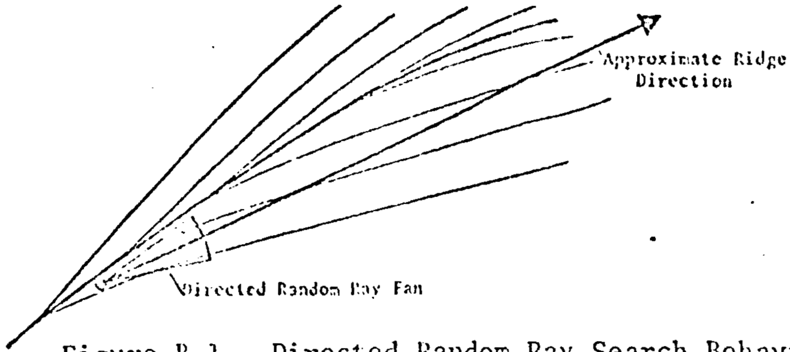

Figure B.1. Directed Random Ray Search Behavior Along a Ridge

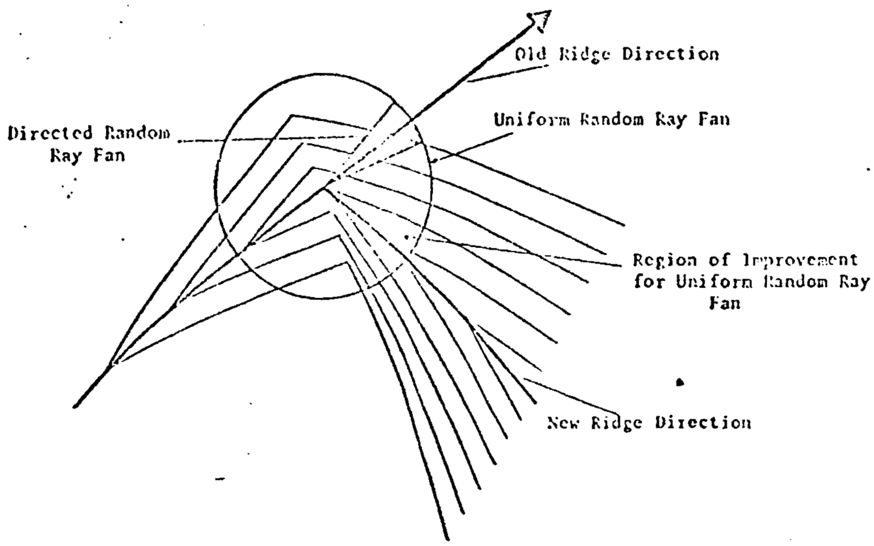

Figure B.2. Behavior of Directed Random Ray Search at Abrupt Ridge Direction Change

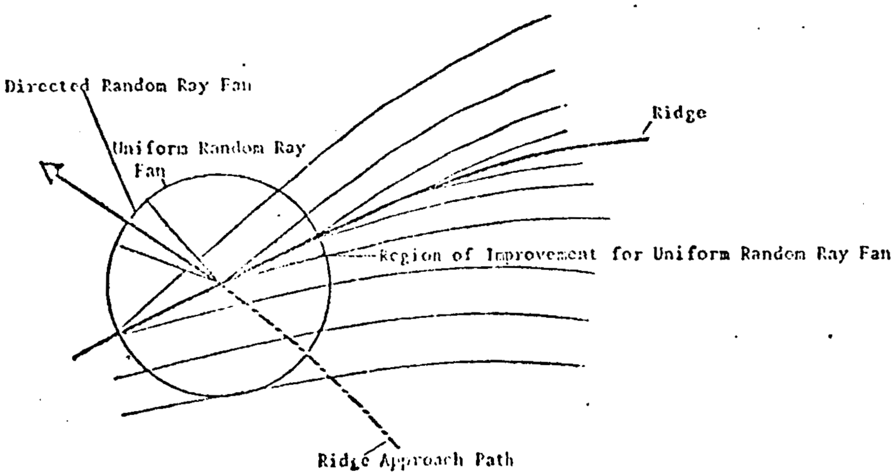

Figure B.3. Behavior of Directed Random Ray Search on Meeting an Inclined Ridge

APPENDIX C

WEIGHTING FACTOR CONSTANTS USED FOR THE SAMPLE CASE OF THE 15,000 LBF ENGINE

$A_{FI} = 150.0$	Constant of the performance characteristic in the rating equation.
$A_{FII} = 200.0$	Constant of the stability characteristic in the rating equation.
$A_{FIII} = .00069$	Constant of the pressure drop characteristic in the rating equation.
$A_{FIV} = .0395$	Constant of the injector complexity characteristic in the rating equation.
$A_{FV} = 40.5$	Constant of the chamber length characteristic
$A_{FVI} = 35.0$	Constant of the chamber diameter characteristic
$A_{FVII} = 66.0$	Constant of the mixture ratio characteristic in the rating equation.
$B_{FI} = 1.0$	Exponential on the performance characteristic in the rating equation.
$B_{FII} = 0.0$	Exponential on the stability characteristic in the rating equation.
$B_{FIII} = 1.8$	Exponential on the pressure drop characteristic in the rating equation.
$B_{FIV} = 2.0$	Exponential on the injector complexity characteristic in the rating equation.
$B_{FV} = 1.86$	Exponential on the chamber length characteristic
$B_{FVI} = 2.0$	Exponential on the chamber diameter characteristic

$B_{FVII} = 1.16$	Exponential on the mixture ratio characteristic in the rating equation.
$C_{FII} = .1$	Exponential on the stability characteristic in the rating equation.
$a_{f11} = 0.0$	Constant in the performance characteristic equation.
$a_{f12} = 1.0$	Constant in the performance characteristic equation.
$a_{f13} = 1.0$	Constant in the performance characteristic equation.
$a_{f14} = .01$	Constant in the performance characteristic equation.
$a_{f15} = .01$	Constant in the performance characteristic equation.
$af_{20} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f21} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f22} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f23} = 0.0$	Constant in the combustor stability characteristic equation.
$a_{f24} = 0.0$	Constant in the combustor stability characteristic equation.
$a_{f25} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f26} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f27} = 1.0$	Constant in the combustor stability characteristic equation.

$a_{f28} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f29} = 1.0$	Constant in the combustor stability characteristic equation.
$a_{f31} = 2.0$	Constant in the fuel pressure drop characteristic equation.
$a_{f32} = 1.0$	Constant in the oxidizer pressure drop characteristic equation
$a_{f41} = .009$	Constant for the injector orifice number.
$a_{f42} = .125$	Constant for the oxidizer dome volume.
$a_{f43} = .125$	Constant for the fuel dome volume
$a_{f44} = 5.625$	Constant for the length of the oxidizer orifices.
$a_{f45} = 4.16$	Constant for the length of the fuel orifices.
$a_{f46} = 0.0$	Constant for the injector type complexity.
$a_{f51} = 1.0$	Constant for the chamber length characteristic
$a_{f52} = 1.0$	Exponential on the chamber length characteristic
$a_{f61} = 1.0$	Constant for the chamber diameter characteristic
$a_{f62} = 1.0$	Exponential on the chamber diameter characteristic
$a_{f71} = 4.0$	Constant for the mixture ratio characteristic.
$b_{f71} = 5.06$	Constant for the mixture ratio characteristic.
$a_{f72} = 2.0$	Exponential on the mixture ratio characteristic equation.